A Reduced-Order Model for Thermal Radiative Transfer Problems Based on Multilevel Quasidiffusion Method

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M&C
August 26, 2019

Outline

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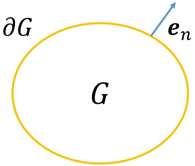
Introduction

- Thermal radiative transfer (TRT) is an essential piece in many multi-physical phenomena whose fields of application include astrophysics and inertial confinement fusion
- The TRT problem encompasses the fundamental features of radiative hydrodynamic problems
  - High dimensionality
  - Multiple scales in time and space
  - Strong nonlinearity & coupling of equations
  - Equations of different types
  - Distinct characteristic behavior for various energy ranges
- The TRT problem creates a valuable test platform for new computational methods before being extended to more complicated physical models

Thermal Radiative Transfer (TRT) Problem

- The radiative transfer (RT) equation

$$\frac{1}{c}\frac{\partial I_g(\mathbf{r},\boldsymbol{\Omega},t)}{\partial t} + \boldsymbol{\Omega}\cdot I_g(\mathbf{r},\boldsymbol{\Omega},t) + \varkappa_g(T)\, I_g(\mathbf{r},\boldsymbol{\Omega},t) = \varkappa_g(T)\, B_g(T)$$



$$\mathbf{r}\in G, \quad \text{for all } \boldsymbol{\Omega}, \quad g=1,\dots,N_g, \quad t\ge t_0$$

$$I_g|_{r\in\partial G} = I_g^{in}, \quad \boldsymbol{\Omega}\cdot\mathbf{e}_n<0, \quad t\ge t_0$$

$$I_g|_{t=t_0} = I_g^0, \quad \mathbf{r}\in G, \quad \text{for all } \boldsymbol{\Omega}$$

$$B_g(T) = \frac{2h}{c^2}\int_{\nu_g}^{\nu_{g+1}} \nu^3\left(e^{\frac{h\nu}{kT}}-1\right)^{-1} d\nu$$

- The material energy balance (MEB) equation gives a basic model of photon interaction with matter

$$\frac{\partial\varepsilon(T)}{\partial t} = \sum_{g=1}^{N_g}\int_{4\pi}\varkappa_g(T)\left(I_g - B_g(T)\right) d\boldsymbol{\Omega}$$

$$T|_{t=t_0} = T^0(\mathbf{r}), \quad \mathbf{r}\in G, \quad g=1,\dots,N_g$$

Reduced-Order Modeling

- The **high dimensionality** of the radiative transport equation motivates the formulation of **reduced-order models**
- To **reduce dimensionality** and create a basis for **reduced-order models** in multiphysics problems involving radiative transfer, it is proposed to formulate a hierarchy of effective low-order transport (ELOT) problems
- These **ELOT problems** can be coupled to other multiphysics equations in a projected space of the **lowest dimensionality** of the system
- We define **ELOT equations** for the angular and energy moments of the specific intensity by means of the **multilevel nonlinear projective-iterative** (MNPI) technique using the Quasidiffusion (QD) method
- The **proper orthogonal decomposition** of the high-order solution and its moments is used to complete the **reduction of dimensionality**

Outline

Multigroup Low-Order QD (MLOQD) Equations

$$\frac{1}{c}\frac{\partial I_g(\boldsymbol{r},\boldsymbol{\Omega},t)}{\partial t} + \boldsymbol{\Omega}\cdot I_g(\boldsymbol{r},\boldsymbol{\Omega},t) + \varkappa_g(T)\, I_g(\boldsymbol{r},\boldsymbol{\Omega},t) = \varkappa_g(T)\, B_g(T)$$

- Projection: $\int_{4\pi}\cdot\, d\boldsymbol{\Omega}$ & $\int_{4\pi}\cdot\, \boldsymbol{\Omega} d\boldsymbol{\Omega}$
- Functions in the projected space: $E_g = \frac{1}{c}\int_{4\pi} I_g d\boldsymbol{\Omega}$ & $\boldsymbol{F}_g = \int_{4\pi}\boldsymbol{\Omega} I_g d\boldsymbol{\Omega}$

$$\frac{\partial E_g}{\partial t} + \nabla\cdot\boldsymbol{F}_g + c\varkappa_g E_g = 4\pi\varkappa_g B_g, \qquad \frac{\partial E_g}{\partial t} + \nabla\cdot\boldsymbol{F}_g + c\varkappa_g E_g = 4\pi\varkappa_g B_g$$

$$\frac{1}{c}\frac{\partial \boldsymbol{F}_g}{\partial t} + c\nabla\cdot\boldsymbol{H}_g + \varkappa_g\boldsymbol{F}_g = 0, \qquad \frac{1}{c}\frac{\partial \boldsymbol{F}_g}{\partial t} + c\nabla\cdot\left(\boldsymbol{f}_g E_g\right) + \varkappa_g\boldsymbol{F}_g = 0$$

- The closure is defined by means of the QD (Eddington) tensor as follows:

$$\boldsymbol{H}_g = \int_{4\pi}\boldsymbol{\Omega}\boldsymbol{\Omega} I_g(\boldsymbol{r},\boldsymbol{\Omega},t)\, d\boldsymbol{\Omega} = \boldsymbol{f}_g E_g, \quad \boldsymbol{f}_g = \frac{\int_{4\pi}\boldsymbol{\Omega}\boldsymbol{\Omega} I_g d\boldsymbol{\Omega}}{\int_{4\pi} I_g(\boldsymbol{r},\boldsymbol{\Omega},t)\, d\boldsymbol{\Omega}}$$

Multigroup Low-Order QD (MLOQD) Equations

$$\frac{1}{c}\frac{\partial I_g(\boldsymbol{r},\boldsymbol{\Omega},t)}{\partial t} + \boldsymbol{\Omega}\cdot I_g(\boldsymbol{r},\boldsymbol{\Omega},t) + \varkappa_g(T)\, I_g(\boldsymbol{r},\boldsymbol{\Omega},t) = \varkappa_g(T)\, B_g(T)$$

- Projection: $\int_{4\pi} \cdot\, d\boldsymbol{\Omega}$ & $\int_{4\pi} \cdot\, \boldsymbol{\Omega} d\boldsymbol{\Omega}$
- Functions in the projected space: $E_g = \frac{1}{c}\int_{4\pi} I_g d\boldsymbol{\Omega}$ & $\boldsymbol{F}_g = \int_{4\pi} \boldsymbol{\Omega} I_g d\boldsymbol{\Omega}$

$$\frac{\partial E_g}{\partial t} + \nabla\cdot\boldsymbol{F}_g + c\varkappa_g E_g = 4\pi\varkappa_g B_g, \qquad \frac{\partial E_g}{\partial t} + \nabla\cdot\boldsymbol{F}_g + c\varkappa_g E_g = 4\pi\varkappa_g B_g$$

$$\frac{1}{c}\frac{\partial \boldsymbol{F}_g}{\partial t} + c\nabla\cdot\boldsymbol{H}_g + \varkappa_g\boldsymbol{F}_g = 0, \qquad \frac{1}{c}\frac{\partial \boldsymbol{F}_g}{\partial t} + c\nabla\cdot(\boldsymbol{f}_g E_g) + \varkappa_g\boldsymbol{F}_g = 0$$

- The closure is defined by means of the QD (Eddington) tensor as follows:

$$\boldsymbol{H}_g = \int_{4\pi} \boldsymbol{\Omega}\boldsymbol{\Omega} I_g(\boldsymbol{r},\boldsymbol{\Omega},t)\, d\boldsymbol{\Omega} = \boldsymbol{f}_g E_g, \quad \boldsymbol{f}_g = \frac{\int_{4\pi}\boldsymbol{\Omega}\boldsymbol{\Omega} I_g d\boldsymbol{\Omega}}{\int_{4\pi} I_g(\boldsymbol{r},\boldsymbol{\Omega},t)\, d\boldsymbol{\Omega}}$$

Grey Low-Order QD (GLOQD) Equations

$$\frac{\partial E_g}{\partial t} + \nabla \cdot \boldsymbol{F}_g + c\varkappa_g(T)E_g = 4\pi\varkappa_g(T)B_g(T)$$
$$\frac{1}{c}\frac{\partial \boldsymbol{F}_g}{\partial t} + c\nabla\cdot(\mathbf{f}_g E_g) + \varkappa_g(T)\boldsymbol{F}_g = 0$$

- Projection: $\sum_{g=1}^{N_g} \cdot \quad \left(\int_0^\infty \cdot \, d\nu\right)$
- Functions in the projected space: $\bar{E} = \sum_{g=1}^{N_g} E_g$ & $\bar{\boldsymbol{F}} = \sum_{g=1}^{N_g} \boldsymbol{F}_g$

$$\frac{\partial \bar{E}}{\partial t} + \nabla\cdot\bar{\boldsymbol{F}} + c\sum_{g=1}^{N_g}\varkappa_g E_g = c\bar{\varkappa}_B a_R T^4, \qquad \frac{\partial \bar{E}}{\partial t} + \nabla\cdot\bar{\boldsymbol{F}} + c\bar{\varkappa}_E\bar{E} = c\bar{\varkappa}_B a_R T^4$$

$$\frac{1}{c}\frac{\partial \bar{\boldsymbol{F}}}{\partial t} + c\nabla\cdot\sum_{g=1}^{N_g}\mathbf{f}_g E_g + \sum_{g=1}^{N_g}\varkappa_g\boldsymbol{F}_g = 0, \qquad \frac{1}{c}\frac{\partial \bar{\boldsymbol{F}}}{\partial t} + c\nabla\cdot\left(\bar{\mathbf{f}}\bar{E}\right) + \bar{\varkappa}_R\bar{\boldsymbol{F}} + \bar{\eta}\bar{E} = 0$$

$$\frac{\partial \varepsilon(T)}{\partial t} = c\sum_{g=1}^{N_g}\varkappa_g E_g - c\bar{\varkappa}_B a_R T^4, \qquad \frac{\partial \varepsilon(T)}{\partial t} = c\bar{\varkappa}_E\bar{E} - c\bar{\varkappa}_B a_R T^4$$

Grey Low-Order QD (GLOQD) Equations

$$\frac{\partial E_g}{\partial t} + \nabla \cdot \boldsymbol{F}_g + c\varkappa_g(T)E_g = 4\pi\varkappa_g(T)B_g(T)$$

$$\frac{1}{c}\frac{\partial \boldsymbol{F}_g}{\partial t} + c\nabla \cdot (\boldsymbol{f}_g E_g) + \varkappa_g(T)\boldsymbol{F}_g = 0$$

- Projection: $\sum_{g=1}^{N_g} \cdot \quad \left(\int_0^\infty \cdot \, d\nu\right)$
- Functions in the projected space: $\bar{E} = \sum_{g=1}^{N_g} E_g$ & $\bar{\boldsymbol{F}} = \sum_{g=1}^{N_g} \boldsymbol{F}_g$

$$\frac{\partial \bar{E}}{\partial t} + \nabla \cdot \bar{\boldsymbol{F}} + c\sum_{g=1}^{N_g} \varkappa_g E_g = c\bar{\varkappa}_B a_R T^4, \qquad \frac{\partial \bar{E}}{\partial t} + \nabla \cdot \bar{\boldsymbol{F}} + c\bar{\varkappa}_E \bar{E} = c\bar{\varkappa}_B a_R T^4$$

$$\frac{1}{c}\frac{\partial \bar{\boldsymbol{F}}}{\partial t} + c\nabla \cdot \sum_{g=1}^{N_g} \boldsymbol{f}_g E_g + \sum_{g=1}^{N_g} \varkappa_g \boldsymbol{F}_g = 0, \qquad \frac{1}{c}\frac{\partial \bar{\boldsymbol{F}}}{\partial t} + c\nabla \cdot \left(\bar{\boldsymbol{f}}\bar{E}\right) + \bar{\varkappa}_R \bar{\boldsymbol{F}} + \bar{\boldsymbol{\eta}}\bar{E} = 0$$

$$\frac{\partial \varepsilon(T)}{\partial t} = c\sum_{g=1}^{N_g} \varkappa_g E_g - c\bar{\varkappa}_B a_R T^4, \qquad \frac{\partial \varepsilon(T)}{\partial t} = c\bar{\varkappa}_E \bar{E} - c\bar{\varkappa}_B a_R T^4$$

Outline

Exact TRT Model: Multilevel System of QD Equations

- The high-order RT equation

$$\frac{1}{c}\frac{\partial I_g}{\partial t} + \boldsymbol{\Omega}\cdot I_g + \varkappa_g(T)\, I_g = \varkappa_g(T)\, B_g(T)$$

$$\Downarrow$$

$$\boxed{\mathbf{f}_g \quad C_{e_n,g}}$$

- The multigroup LOQD equations

$$\frac{\partial E_g}{\partial t} + \nabla\cdot \boldsymbol{F}_g + c\varkappa_g(T)\, E_g = 4\pi\varkappa_g(T)\, B_g(T)$$

$$\frac{1}{c}\frac{\partial \boldsymbol{F}_g}{\partial t} + c\nabla\cdot(\boldsymbol{f}_g E_g) + \varkappa_g(T)\,\boldsymbol{F}_g = 0$$

$$\Downarrow$$

$$\boxed{\bar{\mathfrak{f}} \quad \bar{C}_{e_n} \quad \bar{\varkappa}_E \quad \bar{\varkappa}_{ros} \quad \boldsymbol{\eta}}$$

- The grey LOQD equations coupled with the MEB equation

$$\frac{\partial \bar{E}}{\partial t} + \nabla\cdot\bar{\boldsymbol{F}} + c\bar{\varkappa}_E\bar{E} = c\bar{\varkappa}_B a_R T^4$$

$$\frac{1}{c}\frac{\partial \bar{\boldsymbol{F}}}{\partial t} + c\nabla\cdot\left(\bar{\boldsymbol{f}}\bar{E}\right) + \bar{\varkappa}_R\bar{\boldsymbol{F}} + \bar{\eta}\bar{E} = 0$$

$$\frac{\partial \varepsilon(T)}{\partial t} = c\bar{\varkappa}_E\bar{E} - c\bar{\varkappa}_B a_R T^4$$

ROM: P_1 Approximation

- High-order solution approximated as linear in $\mathbf{\Omega}$

$$I_g = a + \boldsymbol{b} \cdot \mathbf{\Omega}$$
$$\Downarrow$$
$$\boxed{\mathfrak{f}_g = \frac{1}{3}\mathbb{I} \quad C_{e_n,g} = \frac{1}{2}}$$

- The multigroup LOQD equations

$$\frac{\partial E_g}{\partial t} + \nabla \cdot \boldsymbol{F}_g + c\varkappa_g(T) E_g = 4\pi\varkappa_g(T) B_g(T)$$
$$\frac{1}{c}\frac{\partial \boldsymbol{F}_g}{\partial t} + c\nabla \cdot \left(\frac{1}{3} E_g\right) + \varkappa_g(T) \boldsymbol{F}_g = 0$$
$$\Downarrow$$
$$\boxed{\bar{\mathfrak{f}} = \frac{1}{3}\mathbb{I} \quad \bar{C}_{e_n} = \frac{1}{2} \quad \bar{\varkappa}_E \quad \bar{\varkappa}_{ros} \quad \boldsymbol{\eta}}$$

- The grey LOQD equations coupled with the MEB equation

$$\frac{\partial \bar{E}}{\partial t} + \nabla \cdot \bar{\boldsymbol{F}} + c\bar{\varkappa}_E \bar{E} = c\bar{\varkappa}_B a_R T^4$$
$$\frac{1}{c}\frac{\partial \bar{\boldsymbol{F}}}{\partial t} + c\nabla \cdot \left(\frac{1}{3}\bar{E}\right) + \bar{\varkappa}_R \bar{\boldsymbol{F}} + \bar{\eta}\bar{E} = 0$$

$$\frac{\partial \varepsilon(T)}{\partial t} = c\bar{\varkappa}_E \bar{E} - c\bar{\varkappa}_B a_R T^4$$

ROM: $P_{1/3}$ Approximation

- High-order solution approximated as linear in $\boldsymbol{\Omega}$

$$I_g = a + \boldsymbol{b} \cdot \boldsymbol{\Omega}$$
$$\Downarrow$$
$$\boxed{\mathbf{f}_g = \frac{1}{3}\mathbb{I} \quad C_{e_n,g} = \frac{1}{2}}$$

- The multigroup LOQD equations with weighted $\frac{\partial \boldsymbol{F}_g}{\partial t}$

$$\frac{\partial E_g}{\partial t} + \nabla \cdot \boldsymbol{F}_g + c\varkappa_g(T)E_g = 4\pi\varkappa_g(T)B_g(T)$$
$$\frac{1}{3c}\frac{\partial \boldsymbol{F}_g}{\partial t} + c\nabla \cdot \left(\frac{1}{3}E_g\right) + \varkappa_g(T)\boldsymbol{F}_g = 0$$
$$\Downarrow$$
$$\boxed{\bar{\mathfrak{f}} = \frac{1}{3}\mathbb{I} \quad \bar{C}_{e_n} = \frac{1}{2} \quad \bar{\varkappa}_E \quad \bar{\varkappa}_{ros} \quad \boldsymbol{\eta}}$$

- The grey LOQD equations coupled with the MEB equation

$$\frac{\partial \bar{E}}{\partial t} + \nabla \cdot \bar{\boldsymbol{F}} + c\bar{\varkappa}_E\bar{E} = c\bar{\varkappa}_B a_R T^4$$
$$\frac{1}{3c}\frac{\partial \bar{\boldsymbol{F}}}{\partial t} + c\nabla \cdot \left(\frac{1}{3}\bar{E}\right) + \bar{\varkappa}_R\bar{\boldsymbol{F}} + \bar{\eta}\bar{E} = 0$$

$$\frac{\partial \varepsilon(T)}{\partial t} = c\bar{\varkappa}_E\bar{E} - c\bar{\varkappa}_B a_R T^4$$

ROM: Diffusion Approximation

- High-order solution approximated as linear in $\mathbf{\Omega}$

$$I_g = a + \boldsymbol{b} \cdot \mathbf{\Omega}$$

$$\Downarrow$$

$$\boxed{\mathfrak{f}_g = \frac{1}{3}\mathbb{I} \quad C_{e_n,g} = \frac{1}{2}}$$

- The multigroup LOQD equations with $\frac{\partial \boldsymbol{F}_g}{\partial t} = 0$

$$\frac{\partial E_g}{\partial t} + \nabla \cdot \boldsymbol{F}_g + c\varkappa_g(T) E_g = 4\pi \varkappa_g(T) B_g(T)$$

$$c\nabla \cdot \left(\frac{1}{3} E_g\right) + \varkappa_g(T) \boldsymbol{F}_g = 0$$

$$\Downarrow$$

$$\boxed{\bar{\mathfrak{f}} = \frac{1}{3}\mathbb{I} \quad \bar{C}_{e_n} = \frac{1}{2} \quad \bar{\varkappa}_E \quad \bar{\varkappa}_{ros} \quad \boldsymbol{\eta}}$$

- The grey LOQD equations coupled with the MEB equation

$$\frac{\partial \bar{E}}{\partial t} + \nabla \cdot \bar{\boldsymbol{F}} + c\bar{\varkappa}_E \bar{E} = c\bar{\varkappa}_B a_R T^4$$

$$c\nabla \cdot \left(\frac{1}{3}\bar{E}\right) + \bar{\varkappa}_R \bar{\boldsymbol{F}} + \bar{\eta}\bar{E} = 0$$

$$\frac{\partial \varepsilon(T)}{\partial t} = c\bar{\varkappa}_E \bar{E} - c\bar{\varkappa}_B a_R T^4$$

Outline

The Proper Orthogonal Decomposition (POD)

- Cast a problem solution into matrix form $\mathbf{A} \in \mathbb{R}^{m,n}$
  - The n^{th} column holds the spatial vector of solutions for the n^{th} instant of time
  - $\text{rank}(\mathbf{A}) = k = \min(m, n)$
- A singular value decomposition (SVD) represents the matrix in the form

$$\mathbf{A} = \mathbf{U\Sigma V}^T ,$$

  - $\mathbf{U} \in \mathbb{R}^{m,k}$ holds the left singular vectors in its columns
  - $\mathbf{V} \in \mathbb{R}^{n,k}$ holds the right singular vectors in its columns
  - $\mathbf{\Sigma} \in \mathbb{R}^{k,k}$ holds the k singular values (σ) along its diagonal in descending order
- $\mathbf{A}$ can be approximated as a matrix of rank $r < k$ by reducing the dimension $k \rightarrow r$ in its SVD (using the first r singular values).

$$\tilde{\mathbf{A}}^r = \mathbf{U}^r\mathbf{\Sigma}^r\left(\mathbf{V}^r\right)^T ,$$

$$\mathbf{\Sigma}^r \in \mathbb{R}^{r,r}, \quad \mathbf{U}^r \in \mathbb{R}^{m,r}, \quad \mathbf{V}^r \in \mathbb{R}^{n,r} .$$

Applying the POD to Group QD Factors

We derive a parameterized ROM for TRT problems by applying the POD to the group QD factors

- Given the high order solution to some TRT system defined with the set of parameters $\mathcal{L}^{(1)}$

$$I_g^{(1)}, \quad \mathcal{L}^{(1)} = \left\{ I_g^{(1)}|_{r\in\partial G} = I_g^{(1)\text{in}}, \Delta t^{(1)}, \dots \right\}$$

- The QD factors $I_g^{(1)} \rightarrow \boldsymbol{f}_g^{(1)}$ are cast in SVD form

$$\boldsymbol{f}_g^{(1)} = \mathbf{U}_g^{(1)} \mathbf{\Sigma}_g^{(1)} \mathbf{V}_g^{T(1)}$$

- A set of reduced rank QD factors $\tilde{\boldsymbol{f}}_g^{(1)}$ are found from the SVD
- A new problem can be defined with parameters $\mathcal{L}^{(2)}$ and solved with $\tilde{\boldsymbol{f}}_g^{(1)}$ to avoid using the RT equation

Multigroup LOQD ROM: Multilevel LOQD Eqs. with POD

- Compressed representation of the high-order solution as POD expanded group QD factors

$$\Downarrow$$

$$\boxed{\tilde{\mathfrak{f}}_g \quad C_{e_n,g}}$$

- The multigroup LOQD equations

$$\frac{\partial E_g}{\partial t} + \nabla \cdot \boldsymbol{F}_g + c\varkappa_g(T)E_g = 4\pi\varkappa_g(T)B_g(T)$$

$$\frac{1}{c}\frac{\partial \boldsymbol{F}_g}{\partial t} + c\nabla \cdot \left(\tilde{\mathfrak{f}}_g E_g\right) + \varkappa_g(T)\boldsymbol{F}_g = 0$$

$$\Downarrow$$

$$\boxed{\bar{\mathfrak{f}} \quad \bar{C}_{e_n} \quad \bar{\varkappa}_E \quad \bar{\varkappa}_{ros} \quad \boldsymbol{\eta}}$$

- The grey LOQD equations coupled with the MEB equation

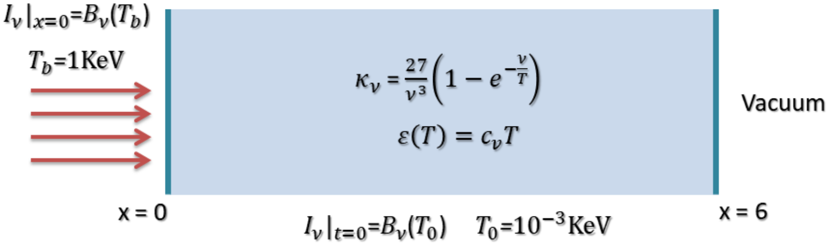
$$\frac{\partial \bar{E}}{\partial t} + \nabla \cdot \bar{\boldsymbol{F}} + c\bar{\varkappa}_E\bar{E} = c\bar{\varkappa}_B a_R T^4$$

$$\frac{1}{c}\frac{\partial \bar{\boldsymbol{F}}}{\partial t} + c\nabla \cdot \left(\bar{\mathfrak{f}}\bar{E}\right) + \bar{\varkappa}_R\bar{\boldsymbol{F}} + \bar{\eta}\bar{E} = 0$$

$$\frac{\partial \varepsilon(T)}{\partial t} = c\bar{\varkappa}_E\bar{E} - c\bar{\varkappa}_B a_R T^4$$

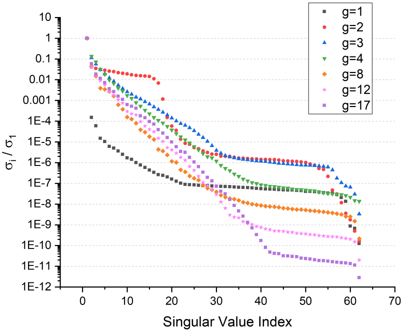
Outline

1D F-C Test Problem



- Fleck & Cummings, 1971
- 17 frequency (energy) groups
- 60 spatial cells, $\Delta x = 0.1$ cm
- $\Delta t = 2 \times 10^{-2}$ ns
- $0 \leq t \leq 6$ ns, 300 time steps
- DS_4 Gaussian quadrature set
- Finite volume in space & fully implicit scheme for LOQD eqs.

QD Factor Analysis



Singular values $(\sigma_{i,g}/\sigma_{1,g})$

$\mathbf{A} \in \mathbb{R}^{m,n}$, $m = 62,\ n = 300$

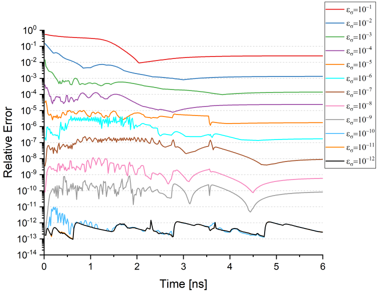
$$\frac{\sigma_{n,g}}{\sigma_{1,g}} \geq \varepsilon_\sigma \quad \text{for all } n \leq r_g$$

Low-rank approximation in energy groups depending on ε_σ

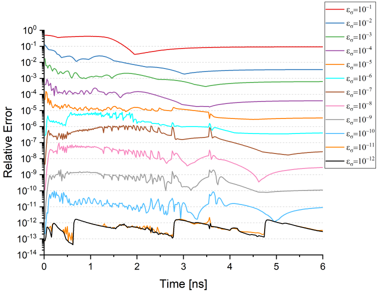
$\varepsilon_\sigma \setminus g$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10^{-1}	1	1	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1
10^{-2}	1	8	6	5	5	4	3	3	3	3	3	3	3	4	4	4	4
10^{-3}	1	18	14	11	10	7	7	7	7	7	7	8	8	8	8	9	9
10^{-4}	2	19	21	17	16	14	12	11	11	12	12	12	13	13	14	14	14
10^{-12}	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62

Solution of the Multigroup ROM

- The F-C test ($\Delta t = 2 \times 10^{-2}$ ns) is solved for the high-order solution and formed into reduced-rank QD factors $\tilde{\boldsymbol{f}}_g$
- The same test is solved with the multigroup LOQD ROM using $\tilde{\boldsymbol{f}}_g$
- Shown is the Relative error in the ∞-norm of solutions of the multigroup LOQD ROM computed for different ε_σ values.



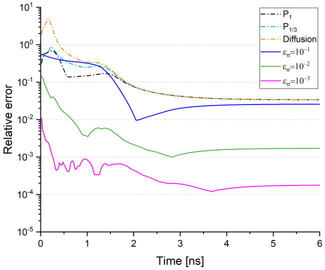
Temperature



Energy density

Multigroup LOQD ROM vs Multigroup Diffusion, P_1, and $P_{1/3}$

- The F-C test ($\Delta t = 2 \times 10^{-2}$ ns) is also solved with classical ROMs such as multigroup diffusion, P_1, and $P_{1/3}$
- Shown is the Relative error in the ∞-norm of solutions of these ROMs compared to the multigroup LOQD ROM



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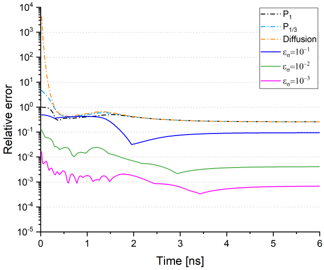
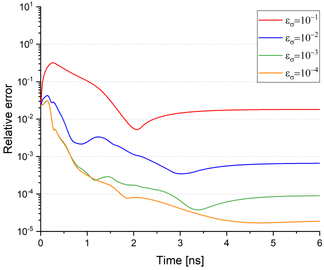


Energy density

Incomplete QD Factor Database: Coarse Time Steps

- The group QD factor database generated for $\Delta t = 2 \times 10^{-2}$ ns is used with the multigroup LOQD ROM to solve the F-C test with $\Delta t = 1 \times 10^{-2}$ ns
- Incomplete portions of the database are calculated using linear interpolation between known values in the database
- Shown is the Relative error in the L_1-norm of the multigroup LOQD ROM



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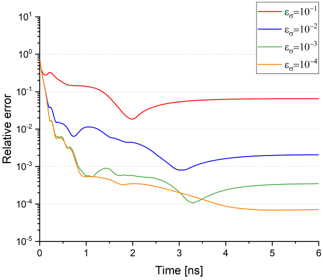
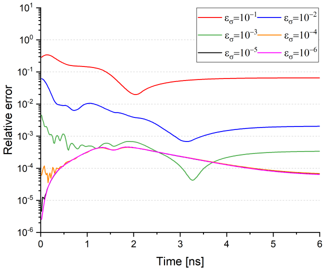


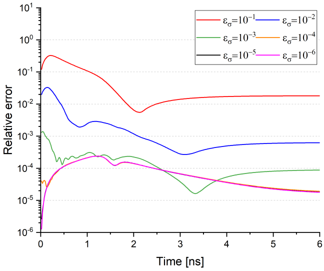
Energy density

Incomplete QD Factor Database: Parameterization

- QD factor databases were formed for $T_i^{(1)}n = 1$ KeV and $T_i^{(2)}n = 0.92$ KeV
- The multigroup LOQD ROM linearly interpolates between these databases to solve the F-C test with $T_i n = 0.96$ KeV
- Shown is the Relative error in the L_1-norm of the multigroup LOQD ROM



Temperature



Energy density

Conclusions

Results

- In this study we developed a novel general methodology for developing reduced-order models for TRT problems.
- The multigroup LOQD ROM sufficiently accurately approximates the solution of the considered TRT problem.
- The multigroup LOQD ROM was shown to have potential in parametric model reduction for TRT problems

Future Work

- Currently a grey (single-group) LOQD ROM is being developed for the same TRT problems
- Future study of the multigroup LOQD ROM includes
  - Refining how the QD factor database is decomposed and compressed
  - Extending the ROM into 2-D
  - Extending the ROM for radiation hydrodynamic problems