Multilevel Method for Thermal Radiative Transfer Problems with Method of Long Characteristics for the Boltzmann Transport Equation

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Introduction

- We investigate the performance of computational methods for high-energy density physics (HEDP) simulations based on ray-tracing schemes (RTS).
- The Boltzmann transport equation (BTE) models the involved radiation transport component in HEDP simulations.
- The method of long characteristics (MOLC) solves the integral form of the BTE over characteristics extending over the entire spatial domain, and does not interpolate the solution on cell faces.
- Effects of using RTS are investigated in the context of generating closures for moment equations which serve to propagate radiation transport effects to multiphysics equations
- Since the BTE solution is defined on a separate discrete grid from that of the multiphysics equations it is coupled with, either grid can be refined or coarsened independently to manage computational resources.

Thermal Radiative Transfer

- The computational methods are analyzed with the thermal radiative transfer (TRT) problem defined with the BTE:

$$\frac{1}{c}\frac{\partial I_g}{\partial t} + \boldsymbol{\Omega}\cdot\boldsymbol{\nabla} I_g + \varkappa_g(T) I_g = \varkappa_g(T) B_g(T),\ \ I_g|_{\boldsymbol{r}\in\partial\Gamma^-} = I_g^{\text{in}},\ \ I_g|_{t=0} = I_g^0$$

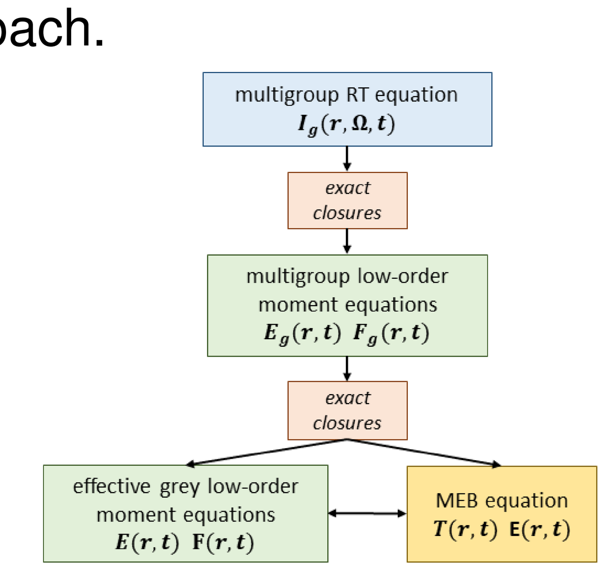
- and material energy balance (MEB) equation:

$$\frac{\partial\varepsilon(T)}{\partial t} = \sum_{g=1}^{G}\left(\int_{4\pi} I_g d\Omega - B_g(T)\right)\varkappa_g(T), \quad T|_{t=0} = T^0.$$

- Specific radiation intensity: $I_g(\boldsymbol{r},\boldsymbol{\Omega},t)$, material temperature: $T(\boldsymbol{r},t)$

Multilevel QD (VEF) Method

- The multilevel quasidiffusion (MLQD) method is formulated by a hierarchy of moment equations derived by projection operator approach.



$$E_g = c^{-1}\int_{4\pi} I_g d\Omega$$

$$\mathbf{F}_g = \int_{4\pi} \boldsymbol{\Omega} I_g d\Omega$$

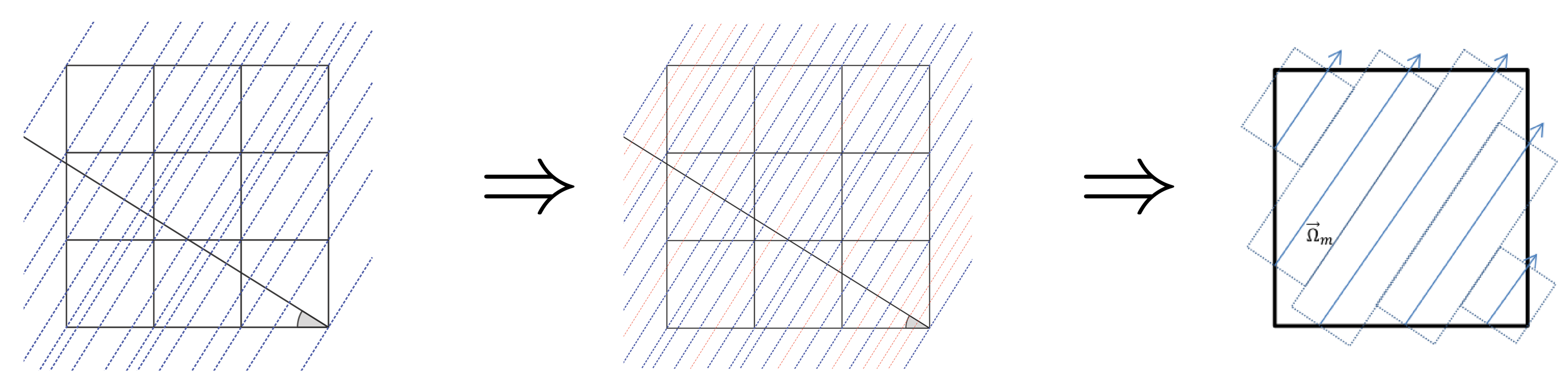
$$E = \sum_g E_g$$

$$\mathbf{F} = \sum_g \mathbf{F}_g$$

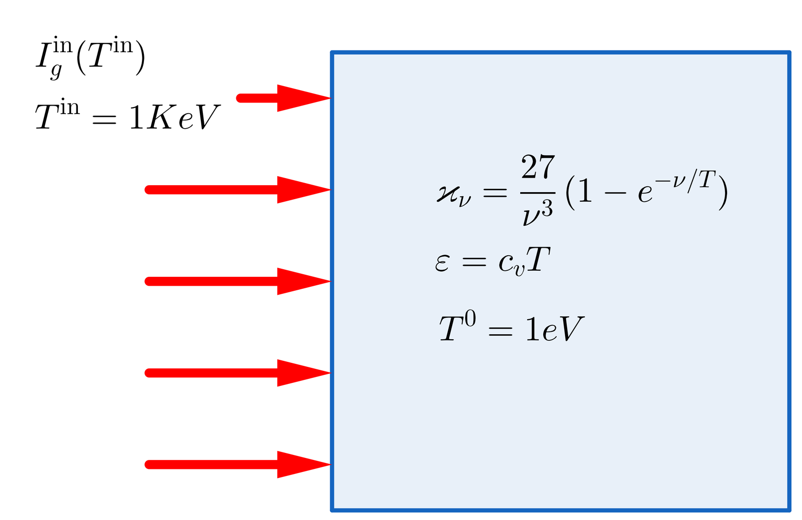
- Exact closures are defined by the Eddington tensor $\mathfrak{f}_g[I] = \frac{\int_{4\pi}\boldsymbol{\Omega}\otimes\boldsymbol{\Omega}\, I_g d\Omega}{\int_{4\pi} I_g d\Omega}$, group-averaged opacities, and other functionals.
- The BTE discretized with the MOLC serves to generate closures for the system of moment equations, which are defined on the same grid as the MEB equation.

Generation of Characteristic Grid

- The material grid is orthogonal, defined for 2D Cartesian geometry
- For each discrete direction of particle motion $\boldsymbol{\Omega}$, a mesh of characteristics (rays) is constructed
- The characteristic mesh is adaptively defined over the material grid to satisfy (i) no characteristic traces over a material cell vertex, (ii) all characteristics have width smaller than h_{moc}.



Numerical Test Problem

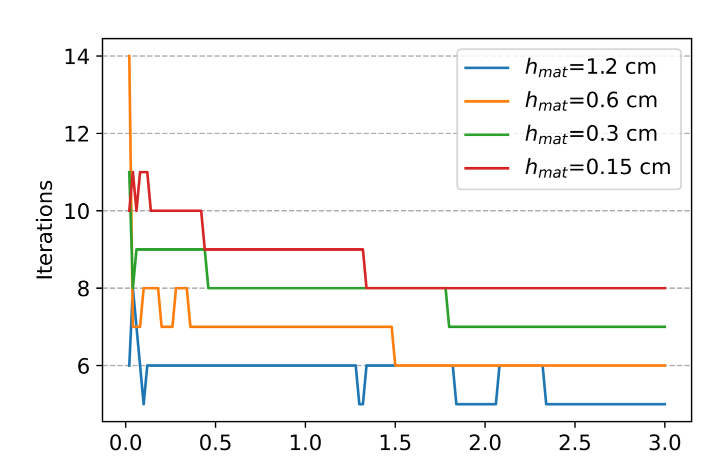


The method is analyzed using the Fleck-Cummings (FC) numerical test problem. It comprises a 6 × 6 cm thick homogeneous domain with $c_v = 0.5917 a_R (T^{\text{in}})^3$. The solution is a supersonic radiation wave which propagates from the left boundary source across to the right boundary. The FC test is simulated for $t = [0, 3]$ ns.

- 150 uniform time steps of length 0.02 ns
- 17 frequency groups, 144 discrete directions
- All equations are discretized with implicit backward-Euler integrator.
- Low-order moment equations are discretized in space with a second-order finite volume scheme.
- The material spatial grid is uniform with cells of width h_{mat}.
- Difference in variable y between mesh refinements ($\|\Delta y_h\|$) and estimated convergence rate (ρ_h^y):

$$\|\Delta y_h\| = \|y_h - y_{2h}\|_{L_2}, \quad \rho_h^y = \frac{\|\Delta y_{2h}\|}{\|\Delta y_h\|}$$

Iterative Behavior



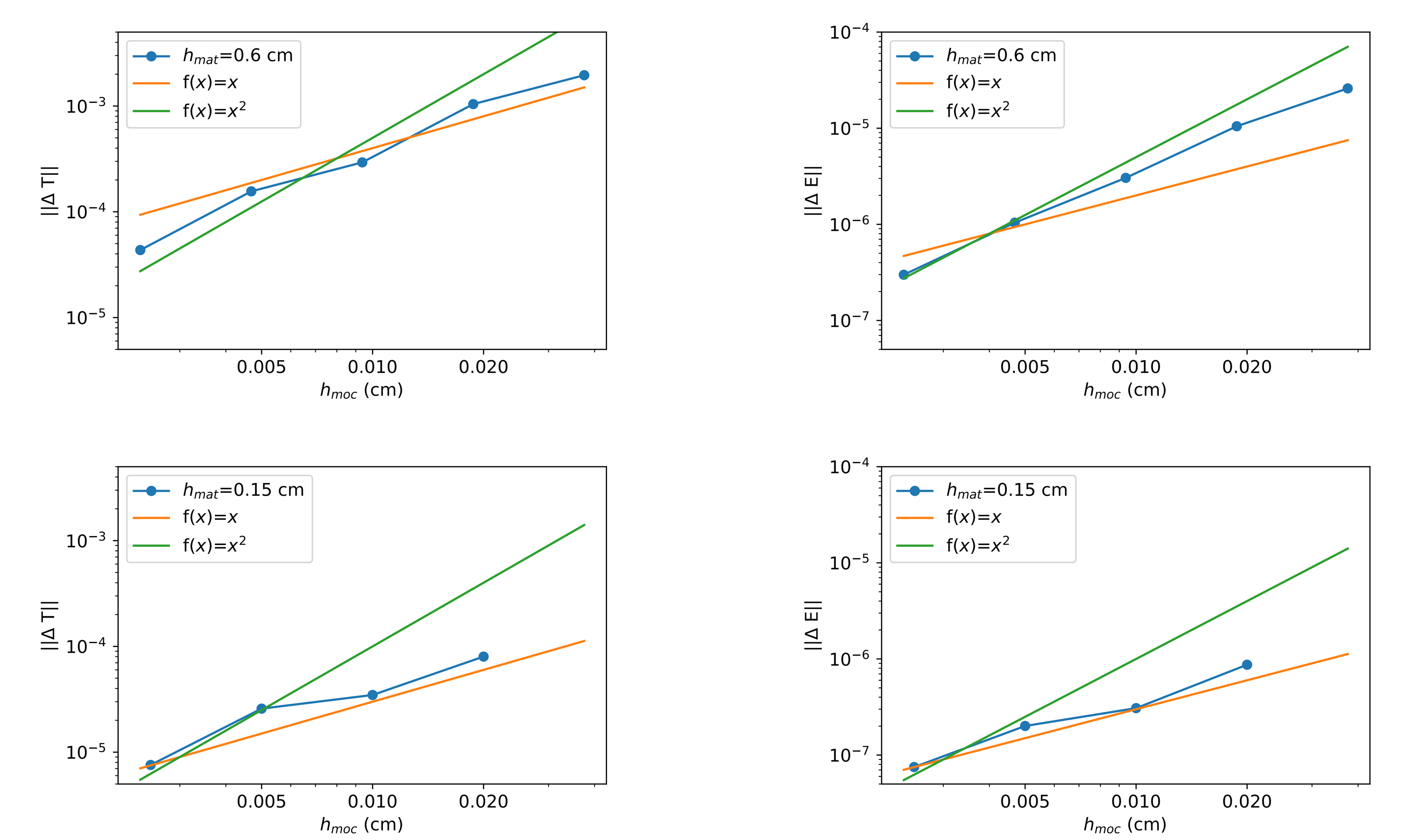
The number of BTE iterations per time step to reach a convergence criteria of $\epsilon = 10^{-14}$ is plotted, with $h_{\text{moc}} = 10^{-3}$ cm and several refinements in h_{mat}. We observe rapid convergence for all cases.

Convergence with Grid Refinements

- Convergence of material temperature (T) and total radiation energy density (E) with refinements in h_{mat} while $h_{\text{moc}} = 10^{-3}$ cm:

h_{mat}	$\|\Delta T_{h_{\text{mat}}}\|$	$\rho^T_{h_{\text{mat}}}$	$\|\Delta E_{h_{\text{mat}}}\|$	$\rho^E_{h_{\text{mat}}}$
0.6	19.1	-	2.13×10^{-1}	-
0.3	9.70	1.96	1.10×10^{-1}	1.94
0.15	4.88	1.99	5.69×10^{-2}	1.97

- Convergence of T and E with refinements in the characteristic grid while $h_{\text{mat}} = 0.6, 0.15$ cm:



Conclusions

- Iterations at each time step are shown to converge rapidly with several material grid refinements.
- Refinement of the material grid yields first order convergence of the solution.
- Refinement of the characteristic grid gives less straightforward convergence behavior. A more detailed analysis of the MOLC/RTS schemes for these problems is warranted to fully understand this convergence behavior.
- The absolute difference between solutions for subsequent refinements in the material grid is shown to be significantly larger than for refinements in the characteristic grid.

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